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I have made an eye-piece on this principle, and the advantage over those in ordinary use is very great. The instrument I have made and tried I have now the honour of exhibiting to the Members of the Society.

Description of a Prismatic Solar Eye-piece.
By John E. Ingpen.

(*Communicated by John Browning.*)

A B C is a prism, having all three surfaces polished; the angle at A is about 45°. A ray of sunlight, D E, from the object-glass of the telescope enters the prism perpendicularly to A B, and is totally reflected from E to F, where a large portion of the light and heat is refracted in the direction F I. The rest is reflected to G, and again totally reflected to H, where it emerges perpendicularly to A C, and is viewed by the eye-piece.

The larger the angle B A C (provided it is less than 60°), the greater the amount of light and heat that is got rid of.

This eye-piece is cool and pleasant in use, but the prism must be very perfect both in material and surface; and even then it is doubtful whether the finest definition can be obtained. It may, however, prove useful in certain cases.

On the Diffraction of Object-glasses. By the Hon. J. W. Strutt, M.A., F.R.A.S.

In observing the Sun with a telescope astronomers have to adopt some device in order to obviate the injurious effects of which the intense light and heat would otherwise have on the eye. The most obvious way of doing this would be to contract the aperture of the object-glass, until the amount of light was reduced to within the necessary limit. But, as is well known, such a course cannot be followed without an enormous sacrifice of definition. The image in the focus of the object-glass of a mathematical point, is a patch of light surrounded by rings, the dimensions of the system for a given wave-length varying inversely with the diameter of the lens. If this be reduced by a diaphragm, the patches dilate, those whose centres are within a small distance overlap, and the resolving power of the telescope suffers.

It has occurred to me that the result would be quite otherwise if, instead of the marginal, the central parts of the glass were stopped off, so that the light, coming from the lens to the focus, formed a hollow cone of rays. In this case the peculiar advantage of a large aperture would not be lost, while any imperfections arising from outstanding spherical aberration would be much diminished.

The general dependence of the diffraction phenomena which occur at the focus of a telescope on the aperture and wave-length,

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may be explained without mathematical analysis. Consider the centre of the image given by a well-corrected object-glass, as illuminated by secondary waves coming from every part of it. Since the phases of all these elements agree at the point in question, the illumination is a maximum, and varies as the *square* of the area of the object-glass. Now, take a neighbouring point in the focal plane. If the difference of its distances from the nearest and furthest point of the lens be but a small fraction of the wave-length of the light, the illumination must be sensibly the same as before. The distance which it is necessary to go from the centre, in order that the illumination may be diminished in a given proportion, will evidently vary directly with λ, and inversely with the diameter of the glass. If, then, we cover up the edge of the glass, the image of a point dilates, but no such effect ensues if we obstruct the central parts.

Another point seems to deserve a passing remark. Red glass is, I believe, often used to diminish the solar glare. From an optical point of view, this is the worst that could be chosen, for the ring system being proportional to λ, is the largest for red light. The wave-lengths for the fixed lines, C, D, F, are in the ratio 655 : 486. The substitution of a green blue light for red would, therefore, be equivalent to an enlargement of about one-third in the diameter of the object-glass.

My immediate object in the present communication is, if possible, to induce some astronomer in possession of a good telescope to make a few observations on the defining power of an object-glass provided with a central stop. The subject might be either the Sun himself, or as in Foucault's experiments (Verdet, *Leçons d'Optique Physique*, tom. I. p. 308), a scale illuminated by sunlight, and placed at a sufficient distance. Any results that might be obtained could not fail to interest physicists generally.

I append a mathematical statement of the two most contrasted cases, namely (1), when the object-glass is completely uncovered, (2) when a narrow marginal rim is alone left to act.

Let $d\,\mathrm{I}$ denote the amplitude of the vibration corresponding to a ring of the object-glass ($2\pi \mathrm{R}\, d\mathrm{R}$) at a point whose distance from the centre of the image subtends at the lens an angle θ. Then, if x be $2\pi \frac{\mathrm{R}\sin\theta}{\lambda}$,

$$d\mathrm{I} = 2\pi \mathrm{R}\, d\mathrm{R}\, \mathrm{J}_0(x),$$

Where

$$\mathrm{J}_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\cos\phi)\, d\phi$$

is Bessel's function of order zero.

For the complete aperture from 0 to R, we have,

$$\mathrm{I} = 2\pi \int_0^{\mathrm{R}} \mathrm{R}\, d\mathrm{R}\, \mathrm{J}_0\left(2\pi \frac{\mathrm{R}\sin\theta}{\lambda}\right)$$

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The intensity is represented in the two cases by the squares of $d\mathrm{I}$ and I. The expression for I may be written,

$$\mathrm{I} = \frac{2\pi \mathrm{R}^2}{x^2} \int^x x\, dx\, \mathrm{J}_0(x) \, .$$

Now, denoting differentiation by accents, $\mathrm{J}_0(x)$ satisfies the equation

$$\mathrm{J}_0'' + \frac{1}{x}\mathrm{J}_0' + \mathrm{J}_0 = 0 \, ,$$

whence,

$$\int_0^x x\, dx\, \mathrm{J}_0(x) = -\int_0^x dx \left\{ x \mathrm{J}_0'' + \mathrm{J}_0' \right\}$$

$$= \int_0^x dx \frac{d}{dx} \left\{ x \mathrm{J}_0' \right\} = - x \mathrm{J}_0' \, ,$$

so that,

$$\mathrm{I} = -\pi \mathrm{R}^2 \, . \, \frac{2\mathrm{J}_0'(x)}{x}$$

or, since,

$$\mathrm{J}_0'(x) = -\mathrm{J}_1(x)$$

$$\mathrm{I} = \pi \mathrm{R}^2 \, . \, \frac{2\mathrm{J}_1(x)}{x}$$

The intensities in the two cases are accordingly in the ratio

$$\left[\frac{2\mathrm{J}_1(x)}{x}\right]^2 : \left[\mathrm{J}_0(x)\right]^2 .$$

Let us consider, first, the positions of the dark rings. Their radii are given by the roots of the equations $\mathrm{J}_1(x) = 0$ and $\mathrm{J}_0(x) = 0$.

In case (2) ($\mathrm{J}_0(x) = 0$) the values of x, corresponding to the dark rings, are

2·41, 5·52, 8·66, 11·8, 14·9, 18·1, &c.,

being ultimately of the form $\left(m - \frac{1}{4}\right)\pi$, for the mth dark ring. These are for the pattern given by the marginal rim alone. For the whole lens we require the roots of $\mathrm{J}_1(x) = 0$, which are

3·83, 7·02, 10·17, 13·3, 16·5, 19·6, &c.

the mth being ultimately $\left(m + \frac{1}{4}\right)\pi$. So far the advantage lies entirely with case (2). If the size of the image be regarded as the space included within the first dark ring, the rim used alone gives a smaller image than the whole lens, in the ratio of 2·41 : 3·83, or about 1 : 1·6. From Foucault's experiments (see

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Verdet), it would appear that the effective image does not extend so far as even the first dark circle. Something, however, may depend on the distribution of brightness. The functions $\frac{2\,J_1(x)}{x}$, $J_0(x)$ have their greatest values when $x = 0$, and are then both equal to unity. But the maxima after the first are less for $\frac{2\,J_1(x)}{x}$ than for $J_0(x)$. The position of the maxima of $[J_0(x)]^2$ are given by the equation $J_1(x) = 0$, and are the same as those just found for the vanishing of $\left[\frac{2\,J_1(x)}{x}\right]^2$.

Corresponding to the values of x,

$$3{\cdot}83, \qquad 7{\cdot}02, \qquad 10{\cdot}17, \text{ \&c.},$$

we have for $J_0(x)$ without regard to sign

$$\cdot403, \qquad \cdot300, \qquad \cdot250, \text{ \&c.}$$

The squares of these give the maximum illuminations.

To find when $\left[\frac{2\,J_1(x)}{x}\right]$ is a maximum, we have by differentiation,

$$J_1'(x) - \frac{J_1(x)}{x} = 0$$

But by a property of these functions,

$$J_2 = \frac{1}{x}\,J_1 - J_1',$$

and, therefore, our conditions are simple,

$$J_2(x) = 0.$$

This gives for x approximately,

$$5{\cdot}1, \qquad 8{\cdot}4, \qquad 11{\cdot}6, \text{ \&c.},$$

the series being ultimately the same as for $J_0(x)$, so that the $(m-1)$th root is $\left(m - \frac{1}{4}\right)\pi$.

The corresponding values of $\frac{2\,J_1(x)}{x}$, without regard to sign, are

$$\cdot13, \qquad \cdot064, \qquad \cdot040, \text{ \&c.}$$

For convenience of comparison I have drawn up a table of the maximum intensities, and the places where the maxima and minima occur for the two cases. The calculations were made by means of the tables of Bessel's functions computed by Hansen. (*Lommel, Studien ueber die Bessel'schen Functionen*).

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	Whole Lens.		Marginal Rim.	
	x	Intensity.	x	Intensity.
1st max.	0·	1	0	1
1st min.	3·83	0	2·41	0
2nd max.	5·1	·017	3·83	·16
2nd min.	7·02	0	5·52	0
3rd max.	8·4	·0041	7·02	·090
3rd min.	10·17	0	8·66	0
4th max.	11·6	·0016	10·17	·057
4th min.	13·3	0	11·8	0

P.S.—Since the above paper was written, Mr. Dunkin has called my attention to a paper by the Astronomer Royal on "The Diffraction of an Annular Aperture" (*Phil. Mag.* Jan. 1841) with which I was previously unacquainted. The purport of the two papers is, however, quite different. My object was not to solve a problem in physical optics, but to direct the attention of the possessors of telescopes to the theoretical consequences of using a central stop, and to suggest that its application may possibly be found advantageous in observations where light would otherwise be in excess. There is no pretence of originality in the mathematics; indeed, the solution of the problem for an annulus is a necessary step in the treatment of the question of a lens with full aperture, the details of which are well known.

On a Volcanic Appearance in the Sun. By M. Chacornac.

(*In a Letter to the President.*)

"J'ai l'honneur de vous communiquer qu'hier le 29 July, vers neuf heures du matin, en faisant l'observation journalière ou habituelle de la grosse tache solaire, qui est dans sa seconde apparition, j'ai aperçu une flamme cratériforme représentant un orifice volcanique en ignition; elle était située sur l'hémisphère sud de l'astre vers le bord au trois quart du rayon, c'est-a-dire, dans une région marginale du disque solaire, occupant a-peu-près le même lieu que l'immense tache noire que nous voyons actuellement.

"Cette protuberance solaire enflammée soutendait un angle d'environ un septième du diamètre solaire et apparu colorée d'une teinte bleuâtre nettement décidée.

"Je ne puis croire à autre phénomène qu'une vision analogue à celle dite du trou D'Ulloa. Je ferai remarquer seulement ici que cette vision s'est produite alors que le Soleil élevé sur l'horison, dépourvu de toute éclipse totale ou partielle, que le ciel était découvert de nuage, celui-ci n'était voilé ou couvert d'aucune écran; ce qui donne à cette observation ou à cette vision un caractère particulier qu'il est de mon devoir de signaler à la physiologie astronomique.

"*Ville Urbienne, le* 30 *Julius*, 1872 (*Rhône*)."